PRICING LIMIT
An analysis of minimum charges for High Limit, Low Premium Cover

By: Morton Lane, Ph.D.

Introduction

Aviation insurers are often asked to provide limits of liability to insureds (a) that are large and (b) whose exposures are difficult to assess. This is particularly true of providers of coverage for product manufacturers and other products liability. The question arises, for several insureds with little or no loss experience, how should such coverage be priced? In particular, is there some minimum price of limit?

The commitment of a limit of coverage obviously exposes the insurer’s capital-providers to potentially large losses. The capital providers will require a return on capital so exposed. One avenue, therefore, for obtaining answers is to look to the capital return implications of large limits. Indeed, answers may be obtained by looking at the returns required in comparable capital markets. That will be the thrust of this first part of the enquiry. Part II will look more directly at the implications for pricing from the scant empirical\(^1\) data that is available.

In short, Part I will be a top down perspective and Part II will seek answers from the bottom up.

To anticipate certain observations, it appears that insurers provide limit to their clients much more cheaply than comparable capital markets. There may be good reasons for the difference, perhaps because the nature of the insurance is different, but those reasons need to be clearly understood. After all, if insurers do not

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Suggested Range of Minimum Rates on Line for Insureds ??

Fig 1

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This paper shall not be considered an offer to sell or the solicitation of an offer to buy securities. All information has been obtained from sources both public and private that are believed to be reliable but the authors make no representation as its ultimate accuracy. The views and opinions are those of the authors and are not intended to guarantee any level of financial performance, risk exposure or investment outcome.
provide limit, insureds will have to go to the capital markets themselves to get contingent coverage and it will apparently cost a great deal more than insurance. Insureds may as a result cut the size of their purchased limit. So, pricing limit too cheaply has two consequences. First, there is a danger that committed capital will not be adequately rewarded and therefore withdrawn. Second, pricing the limit too low can lead to over-purchase by insureds and further overexpose capital.

PART I: Market Comparatives

The capital market analogue to providing an insurance limit is that bond markets regularly extend cash to entities in fixed amounts that may or may not be repaid. When the bonds are not repaid, the lender suffers a loss in the same way that an insurer does when a claim is made. If the borrower is particularly creditworthy the frequency with which such non-repayment will be experienced will be rare. In aviation insurance, experience of limit losses for products manufacturers are equally rare. Bond market and insurance market losses can therefore be very similar in terms of both frequency and severity of loss. The difference between the bond market loss and an insurance loss lies in cash flow. In the bond market, cash is paid up front – loss is the lack of repayment; in insurance no cash is advanced until a claim is made. (Borrowers do not experience a credit risk. Insureds do. It’s called the problem of reinsurance recoverables.)

However, not all capital market instruments advance cash. As in insurance, some are contingent in nature. In particular, the bank market issues standby letters of credit that will be drawn at a later date. The conditions for such drawdown can be unrestricted or conditional but obviously the standby advances cash as needed and may not ultimately be repaid.

As a final analogue certain capital markets activities are themselves insured and an active market in those instruments reveals comparative prices. This takes place in two ways. First, this occurs through using monoline financial guarantee companies such as MBIA and FGIC. Second, surrogate insurance exists through the use of Credit Default Swaps. In short, there is a variety of capital market instruments that may be usefully compared to insurance with low premium and high limit. There follows a review of the prices in several of those markets.

The AAA Corporate Bond Market

The yield of corporate bonds reflects two components: the time value of money as represented by treasury rates, and a premium to compensate the lender for the risk of default. The spread between the yield and the treasury rates represents that premium. The spread for AAA rated 5 year corporate bond obligations for the last ten years is shown in Figure 2.

AAA securities default very infrequently. According to Standard and Poor’s, they default 1
Consider Table 1 detailed in Grants Interest Rate Observer of March 12, 2004. Clearly, the price paid for limit over and above the treasury rate for the GSEs is lowest of the premier private corporate issuers. The two AAA private entities listed in the Table may also be considered “too big to fail”.

<table>
<thead>
<tr>
<th>Issuer</th>
<th>Maturity</th>
<th>Rating</th>
<th>Yield</th>
<th>Spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fannie Mae</td>
<td>2/15/2006</td>
<td>AAA</td>
<td>1.62%</td>
<td>13 bps</td>
</tr>
<tr>
<td>Freddie Mac</td>
<td>2/15/2006</td>
<td>AAA</td>
<td>1.64%</td>
<td>15 bps</td>
</tr>
<tr>
<td>AIG</td>
<td>1/30/2006</td>
<td>AAA</td>
<td>1.62%</td>
<td>13 bps</td>
</tr>
<tr>
<td>GECC</td>
<td>1/30/2006</td>
<td>AAA</td>
<td>1.60%</td>
<td>11 bps</td>
</tr>
<tr>
<td>Citi-Group</td>
<td>3/3/2006</td>
<td>AA-</td>
<td>2.06%</td>
<td>58 bps</td>
</tr>
<tr>
<td>Wells Fargo</td>
<td>3/3/2006</td>
<td>AA-</td>
<td>2.07%</td>
<td>58 bps</td>
</tr>
</tbody>
</table>

Source: Grants Interest Rate Observer March 2004

In insurance terms, AAA securities have a low frequency of loss and high severity of loss.

Despite the low expected loss rate the capital market demands a rate-on-line (spread) as high as 100 basis points and as low as 25 basis points. Clearly, the range of spreads indicates that the economic cycle is an important determinant of price. When capital is scare, spreads are high, and conversely. This is important; notwithstanding shifting perceptions about expected losses, scarcity matters. Should insurance capital be different?

Only on very rare occasions did the rate dip into the low 20’s. So if the comparative market for pricing low frequency insurance events is the AAA corporate market, the minimum price of insurance limit should be 20 basis points. That is the level for the average AAA corporate, but, some AAA credits are better than others. Their spreads will be lower than the average – see below. More comparisons are appropriate.

GSEs – Fannie Mae and Freddie Mac

Lenders to private corporations cannot ordinarily be expected to be bailed out by the government when their borrowers fail to repay. Technically, buyers of the obligations of Government Sponsored Enterprises (GSEs) of which Fannie Mae and Freddie Mac are the most well known, cannot be expected to be bailed out either. However, there is great ambiguity about whether or not such a government guarantee exists. There is a wide-spread feeling that Fannie Mae and Freddie Mac are “too big to fail”. This is reflected in the price that is paid to lenders to the GSEs.

The conclusion to be drawn from Table 1 is that the minimum price of limit is 10 – 15 basis points.

Standby Letters of Credit

Of the non-funded comparisons, private banks issue irrevocable standby letters of credit (“LoCs”) to be drawn down if and when needed by a borrower. The pricing by banks of those facilities reflects several things. First, it will reflect the chances of the line being drawn. Second, it will reflect the assessment that, when drawn, the borrower will eventually pay off the loan. It will also depend on the regulatory capital rules.

Unfortunately, letter of credit prices are not transparent. No organized public market in LoC prices exists. Anecdotal evidence suggests that prices between 15 basis points and 90 basis points may be charged depending on the creditworthiness of the borrower and the economic cycle. Minimum charges are then in the order of 15-20 basis points.

A sketch of the method of calculating can also be instructive. Typically, bank regulators will impose capital charges for different lines of business. For example, the capital charge might be 6%. Now, if the bank requires a return on capital of 15%, then the charge for that line must be 90 basis points (=0.15 x 600bps). Superior
credits will obviously require much less of a capital charge. The point is not the numbers but the linkage between capital and minimum pricing. It shows one way forward for establishing minimum charges in insurance.

**Bond Insurance**

Another comparative market for gauging limit minimums is the bond insurance market itself. Certain issuers of debt, who are not of the highest credit, can improve issuer acceptance by “insuring” their bonds with the so-called monoline insurers (MBIA, Ambac, FGIC, etc.) Essentially, the insurer steps in the shoes of the issuer and agrees to make good the payment of principal, should the issuer fail to pay. The charge for that insurance is another indication of rates that are charged for limit on capital market instruments.

There are two branches of bond insurance. The original market was for insuring municipal bonds, i.e. bonds issued by local governments. The more recent branch is for credit enhancing private issues or private structures. There is a further difference. The practice for municipal bonds is to insure principal and interest in exchange for a single premium, covering default for periods of up to 20 year maturities. Private insurance on the other hand is typically for principal only and premium is charged on an annual basis.

A municipal illustration might be as follows. Consider a municipal 20 year single A credit which would otherwise trade at 4.55%. Insured by a monoline the bond might trade at 4.35%, an issuer gain of some 20 basis points. However, the term structure of rates varies by maturity. A 2 year maturity might only show a pick-up of 11 basis points. Since municipal bond insurance is for interest payments and ultimate payment of principal, the weighted average of that stream of guarantees might be some 14 basis points. Finally, it is generally accepted that the municipal market charges an insurance premium equal to roughly half the benefit of the insurance. So, for very good (single A) credits the minimum charge might be 7 basis points per year. Clearly, a AA credit would pay somewhat less. Note that the present value of the stream of annual payments would form the basis for a single

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<tbody>
<tr>
<td>Gross domestic asset-backed par written (millions)</td>
<td>$49,402</td>
<td>$157,306</td>
<td>$164,845</td>
<td>$110,209</td>
<td>$106,539</td>
<td>$94,132</td>
</tr>
<tr>
<td>Weighted-average premium rate for the period (%)**</td>
<td>0.28%</td>
<td>0.20%</td>
<td>0.17%</td>
<td>0.18%</td>
<td>0.18%</td>
<td>0.17%</td>
</tr>
<tr>
<td>Gross per-period capital charge (%)**</td>
<td>2.49%</td>
<td>1.98%</td>
<td>1.73%</td>
<td>2.06%</td>
<td>2.78%</td>
<td>3.01%</td>
</tr>
<tr>
<td>Profitability index (%)***</td>
<td>11.37%</td>
<td>10.25%</td>
<td>10.07%</td>
<td>8.75%</td>
<td>6.51%</td>
<td>5.57%</td>
</tr>
</tbody>
</table>

*Premium rates on asset-backed are annual amounts based on par insured.

**The capital charge is S&P’s estimate of the worst loss of the portfolio of bonds under review.

***The Profitability Index is premium divided by the capital charge.
profitability of bond insurance is unacceptable in the high single digits. Only when premiums are in the twenties are insurance activities profitable.

The conclusion to be drawn from this comparison is that the minimum rate on line for private structured products is 20+ basis points. The exception is municipal bond insurance where the minimum premium is considerably lower, around 5 basis points. There are reasons for this big difference of between 5 and 20 basis points, but we will return to them in the summary of Part I.

Credit Default Swaps

A final potential comparative market is the relatively new credit default swap (CDS) market. Like bond insurance, a third party, the seller of the CDS, steps into the shoes of the issuer in exchange for an annual premium. The buyer is then protected against default by the issuer because of his CDS purchase. The CDS market is more standardized, but exactly similar in function to bond insurance and is a transparent traded market. The prices listed by Deutsche Bank last September 2003 for several CDSs are listed in the Table 3.

The message from this market is not too different from the other comparative markets – namely that the minimum rate for limit for the best of credits (rarest of risks) is in the low 20+ basis points (or in exceptional cases the mid teens). Notice particularly that the reinsurers themselves, prior to many of last year’s downgrades, would have to pay 20+ basis points for protection. Can they therefore afford to receive less from the limits of insurance they write?

Summarizing the Comparatives – The Message of the Markets

Arranging the data from other markets, the range of premiums is laid out as in Figure 1. What is interesting in this line-up is that the public insurance entities (Municipal bonds and, arguably, GCEs) are to the left and at the lowest rates. Private entities are to the right and are at higher premium levels. Why the difference? Essentially, the public entities have the right to tax. If they default on their obligations, states and municipalities have the right to tax, and more specifically to raise taxes, so as to eventually redeem their obligations. Bond holders may suffer a rescheduling of payments but rarely suffer the total loss that can arise with private entities. GCEs may not have explicit access to taxpayer support but the perception of being “too big to fail” drives premiums in that direction.

If these markets provide an appropriate comparison for pricing limit for insurance, what is the appropriate rate that these markets imply? Aviation insurers do not have the ability to tax their clients to recoup loss, not least of the reasons being that neither they nor the client may survive such a loss – witness all the markets that disappeared after 9/11. That said, however, insurers may have the ability to raise premium rates, post event. Similarly, most aviation insurance is written as part of a multi-line book which can support an aviation loss at least temporarily. They can be said to have the ability to tax other lines.

There is also still at large in the insurance community a view that insurance is a smoothing (loss sharing) versus a risk transfer device. Smoothing losses would argue for low premiums (like the municipal market); risk transfer argues for private market premiums. Then again, even within the “smoothing losses” philosophy, there...
is a question of whether the smoothing should be confined to the loss-receiving entity by some smoothing over time, or shared with all market participants. The private market practice seems clearer cut. The premiums charged are on a “no tears” basis. If the insurer receives a loss, the level of premium is set such that the loser cannot expect any special treatment post-loss. Although, it should be remembered, if losses happen it is likely that all clients will experience a rise in subsequent premiums - witness Figure 1 - as the supply and demand for risk transfer capital shifts.

As long as the dichotomy between the smoothers and risk transferors persists, the range of premiums for pricing limit probably lies in the range of 10-20 basis points.

It is beyond the scope of this paper to recommend how particular clients should view insurance prices and structure their purchases. However, a reasonable strategy might be to accept the smoothing benefits of their own actions via some self insured retention, and to get the benefits of risk transfer via their large limit purchases.

Ultimately, however, the comparative markets can only provide insights. Exact pricing depends on analyzing the numbers and those are detailed in Part II.

PART II: Empirical Analysis

Fortunately for insureds, and even more fortunately for insurers, loss events that exhaust limits of liability are rare. This does not mean, however, that insureds are prepared to forego the comfort of large limits. Instead, they typically require considerable protection and insurers are confronted with the problem of pricing limit when they have little large-loss evidence upon which to base that price. The situation is similar to issues confronting insurers of physical catastrophe. How do you charge for a large hurricane limit attaching at $50 billion loss when there have only been 5 or 6 hurricanes causing losses over $5 billion and none over $25 billion? How do you charge for a $1 billion liability limit when the largest historical event is, say, $380 million? The analytic challenge is the same and in this paper we adopt a variation of the techniques used in catastrophe reinsurance to answer the liability question.

The technique under review is “extreme value theory” (EVT). This methodology owes its parentage to the desire of engineers to build dams and bridges so as to withstand extreme events such as a 1 in 500 year flood, etc. Given a symmetric distribution of outcomes a lack of data on extremely large outcomes could be balanced by observations of small outcomes. Fitting an empirical distribution to the data is, therefore, relatively straight forward. However, with asymmetric distributions there will be more small observations than large outcomes. A different fitting procedure is called for. The EVT solution divides the data into small losses and large losses (defined over some threshold) and fits different distributions to each. This is the so called “peaks over threshold” method. EVT theory asserts that the distribution of peak observations will follow a fat-tailed distribution of the Generalized Pareto distribution type. Fitting the two parts separately will give a better gauge of likely extremes than fitting the data jointly in a single distribution. First, however, the data has to be arranged in a way to make that fitting possible.
Organizing the Data

Liability data is usually generated in triangles arranging underwriting years against calendar year claims. Typically, claims for a particular underwriting year will only emerge over several years. One of the tasks of actuaries is to take those years of claims and gauge how the losses will develop in forthcoming years. To consider the losses for underwriting year 1997, for example, claims will have been received during 1997, 98, 00, 01, 02 and 03. In matters of reserving and in matters of how new 2004 underwritings should be priced it is important to know how that class of business develops. What will be the ultimate loss from that underwriting year? Does the cumulative claim by 2002 represent 80% of the ultimate loss from 1997 activity or does it represent 95% of the loss? This is, of course, a tricky question to answer. Much actuarial science is devoted to the question, but even the most dedicated actuary would also concede that much art is also required. In any case, our purpose here is not to reinvent the actuarial techniques. It is to take data and explore whether it can be used to calculate minimum rates on line for various classes of business. For this it is necessary to calculate not only the level of ultimate loss but also to calculate its volatility, or potential for extreme outcomes.

We start by taking the claims data for each class of business, arranging it in the appropriate triangles and applying standard Bornhuetter-Ferguson (BF) techniques to estimate loss development factors (LDFs). It is, of course, essential that the claims be made on a consistent basis and in the case of aviation insurance it is relatively easy to take line-shares in different years and gross them up to “market” loss estimates. Now the loss development factors so derived can be used to gauge ultimate loss for each underwriting year and line of business.

However, we can go further. Claims in 1997 are made in 1997 dollars; claims in 2003 are made in 2003 dollars. Inflation makes the two dollars unequal. So, in order to put the ultimate loss data on the same basis one year to another, an inflation adjustment is necessary. Once again, science and art are important. The inflation adjustment can be purely monetary or it can include elements of social inflation accounting for the fact that court settlements have tended to creep over time. Other adjustments are desirable as well. In particular, adjustments need to be made for “exposure” changes (positive or negative) over time. Is the fuel that is handled by refuellers more or less flammable than it was several years ago? Is the airport exposed to greater or lesser liability threats than it was five years ago? We are not expert in these answers, nor are we suggesting changes to the way the questions are asked. We simply record that to the greatest extent possible the data should be adjusted in such a way as to bring the data to a consistent, comparable basis. At the end of the day it will be desirable to say that ultimate claims from the underwriting of a set of accounts in 1997 is $X and that is comparable with an estimate of $Y from underwritings in 2001.

It is easy to see where this is going. If ultimate loss levels have been observed of $X, $Y, and $Z, etc., for each underwriting year, and each is on a comparable basis, then rough estimates can be made of the mean and the volatility of ultimate losses for each class of business.

Often that is where the exercise stops. Unfortunately, with only something like 12 years of data available to aviation underwriters the 12 points can be too small to provide comfortable estimates of either the mean or the volatility of a class of business. Some of the issues with the average estimate are clear to see. Should it be a straight average? Should it be trended? Should it only involve the most recent observations? And, if the average is questionable, the estimate of standard deviation is even more limited. It is certainly difficult to get a measure of skewness and the fat-tailed nature of the risk with only 12 observations. In order to get some more insightful measures we disaggregate the data by account.
Disaggregating the Data

Claims are collected by account and by underwriting year. Accordingly, it is possible to adjust account/claims data to put it on a comparable basis in much the same way that the “class of business” data is adjusted above. More precisely, LDFs are calculated for each line of business using the BF method referred to above and are then applied to claims in that line on an account-by-account basis. Line size, inflation and exposure adjustment factors are also applied on a line size basis. The result is a matrix of ultimate losses by account, for each underwriting year under review. As long as the accounts are relatively homogenous we can make the bold assumption that all accounts can be represented by an “average” account. Obviously, the more heterogeneous are the individual accounts the less good is the assumption. However, when that assumption is acceptable it allows an examination of more data than is available in a simple year-by-year summary.

Examining the data in the account-by-year matrix allows estimates to be made of the frequency that an account will have no claims. It is simply the fraction of elements in the matrix that are zero. The remaining (positive) elements can be divided into small claims and large claims, where the threshold between small and large is set at some discretionary level above the median loss. Distribution functions can now be fitted to the small loss and large loss data separately. In the exercises conducted for this paper Inverse Gauss distributions were fitted to all the sample sets. Inverse Gauss fits ranked first, second or third best fits by the usual distribution-fitting metrics.

The average account can now be represented as having a loss in any one year equivalent to a drawing from one of three distributions. First, it could be a claim free year with a probability of, say, 80%. Second, it could be generated from the fitted small-loss distribution with a probability of, say, 12%. Finally, it could be generated from the fitted large-loss distribution with a probability of 8%. EVT theory asserts that because the procedure pays more attention to separately fitting large losses it will better represent the extreme possibilities.

Technically, of course, EVT could be applied to the annual data for each line. However, with only 12 annual observations the fitted curves will not inspire confidence. They may well exaggerate the possible extremes.

The final step for estimating the volatility of a line of business is to simulate a portfolio of “average” accounts. The number of accounts in the portfolio is set equal to the existing number of accounts in each line. It is implicitly assumed that each account’s claim activity is independent of any other. The sum of all the independent synthetic account activity will represent the possible outcomes over, say, 20,000 synthetic years versus 12 observed years. We should note
that this procedure does require a large number of simulations to stabilize the results. Small simulations will not do.

The effect of the different ways in which distributions can be estimated is illustrated in Figure 3. Clearly, if the annual data is used to estimate standard deviation and that is assumed to be symmetric, it will severely underestimate tail risk. On the other hand, three different procedures (assuming the data is log normally distributed, using the disaggregated-by-account data, and using annual data with an EVT approach) all show a significant possibility of claims outcomes larger than have been observed in the 12 years of observed data. As long as the objective of the exercise is to get a good estimate of bad surprises, some EVT-like analysis seems to be essential.

In what follows we have used real data from our initial studies. However, in order to preserve confidentiality, all data is referenced to an undisclosed mean which is represented by an algebraic letter.

(Hypothetical) minimum rates on line for a “stand alone” insurer

Suppose an aviation insurer (Independent Insurer) existed that specialized in, and exclusively underwrote, the liability of airline Ground Handlers. Suppose also that its capital came from the capital markets and that it maintained an investment grade credit rating. Further assume that after conducting a simulation exercise such as that described above it determined that the characteristics of Ground Handlers risk were captured by a loss distribution shown below, and that the aggregate limit required by the Ground Handlers was 378 x $B million.

On average, ultimate losses will equal $B million, but 50% of the time they will be below 0.85 x $B million and 50% above 0.85 x $B million. Further, one year in ten ultimate losses will exceed 1.76 x $B million, etc.

To maintain an investment grade rating the Independent Insurer will need to have sufficient resources to meet its obligations 99% of the time (or even one year in two hundred and fifty years). In other words, it must survive a loss of 3.19 x $B to 3.77 x $B million. Clearly, the premium charged will cover some of the loss but unless that premium is to be prohibitive, part of the rare loss will have to be paid out of capital.

How much capital can be exhausted? If the Independent Insurer is to stay in business post loss, it will have to retain some capital. Assume then that it is decided that no more than 1/3 of capital can be lost; otherwise continuing business will be impossible. With the 2/3 of capital intact it is assumed that Independent Insurer can continue to write new business and, perhaps, enjoy increased post loss premiums.

So, if a 3.19 x $B million loss must come from premium and capital, and if average premium is set (for the purposes of the first step in the illustration) equal to expected losses $B, then 2.19 x $B million (= 3.19 x $B - $B) must come from capital. If that amount must be no more than 1/3 of capital, then capital must be at least 3 x [2.19 x $B million].

However, setting premium exactly to expected loss ($B) will produce expected profits of exactly zero. Capital is not devoted to risky enterprise to garner zero rate of return. Instead, capital will flow to less risky pursuits or to higher rewards available in the capital market. If, instead, premium is set to achieve some minimum rate of return an estimate of minimum rates on line can be established. Independent

Table 4

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<thead>
<tr>
<th>GROUND HANDLERS LOSS PROFILE</th>
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<tr>
<td>000,000’s</td>
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<tr>
<td>MEAN</td>
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<tr>
<td>STD DEV</td>
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<tr>
<td>MEDIAN</td>
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<td>0.9990</td>
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<td>0.9998</td>
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Insurer might target a 20% rate of return before taxes, etc., but it might also require a minimum of 10% as the least acceptable to capital providers.

Following that line of reasoning, a load must be generated on capital. To establish the correct levels of premium and capital we must simultaneously solve for a 10% return on equity and a level of premium that does not bite into capital by more than 1/3 the initial level. Taking this dynamic into account with the appropriate algebra gives the following charges. Minimum premium to achieve a 10% rate of return on equity is 1.51 x $B million. This is composed of an expected loss of $A million plus a load of 0.51 x $B million. The capital necessary to run the business would be 5.07 x $B million. Now, a loss of 3.19 x $B million will provide a capital bite of 1.68 x $B million (=3.19 x $B – 1.51 x $B), exactly 1/3 of 5.04 x $B million, the original capital.

The total limit purchased by Ground Handlers is 378 x $B million. If Independent Insurer only wants a minimum 10% return on equity, before taxes, etc., the minimums it should charge are listed in Table 5 below.

<table>
<thead>
<tr>
<th>Total LIMIT Purchased</th>
<th>Expected Loss</th>
<th>Load for 10% RoE</th>
<th>Total Premium for Minimum</th>
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</thead>
<tbody>
<tr>
<td>378 x $B</td>
<td>$B</td>
<td>0.51 x $B</td>
<td>1.51 x $B</td>
</tr>
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If each client is responsible for the average experience plus a price for limit purchased (assuming minimum pricing is satisfactory to the insurer) then the premium per unit of limit would be 39.8 basis points [=1.51 x $B/378 x $B]. A limit of $100 million would cost a minimum of $398,000; a limit of $1 billion would cost $3,980,000.

If, however, a new client without any loss experience were charged simply on his record then a capacity charge of 13.4 basis points per million of limit is suggested. A limit of $100 million for the “average” loss-free account would be $134,000; a limit of $1 billion would be $1,340,000.

Two things are striking about this number of 13 basis points. First, it is similar to the range of prices that the comparative analysis from Part I produced. Second, it is clear that this “bottom up” calculation would be impossible without an adequate measure of the volatility of losses. Without such a measure proper calculation of prices would be impossible. The capital market comparisons were with AAA credits. The above analysis was for “investment grade” at around the 99 percentile loss point. A AAA analysis would more likely be at the 1:250 levels and may tolerate less of a diminution of capital at the 1:250 loss levels. Assuming that a loss of ¼ of initial capital is acceptable for a AAA to continue in business, the comparative premium to produce a 10% return on capital would be 20.9 basis points. Clearly, ambitions for less than minimum returns would scale up the premiums accordingly.

Table 5

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<tr>
<th>GROUNDHANDLERS MINIMUM RATES ON LINE</th>
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<tr>
<td>Total LIMIT</td>
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<tr>
<td>Expected Loss</td>
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<tr>
<td>Load for 10% RoE</td>
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<tr>
<td>Total Premium for Minimum</td>
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In summary, the range of possible minimum premiums is 13 to 21 basis points for an Independent Insurer standing alone in the capital markets and this would be very similar to any other capital market instrument. But, it will be said, insurance is different. There are no stand alone specialist aviation insurers.
Minimum pricing of a liability book with multiple lines

Most aviation insurance is done by multi-line insurers and most is done by pools of underwriting capital allocated from multi-line reinsurers. In other words, it might not have to be priced as rigorously as a stand alone risk. When multiple lines of insurance are written, the writer gains a diversification benefit. When the insurance is part of a larger multiline company the capital standard is not as great as that required for an independent insurer. The consequence of both will lower the level of premium required for minimum pricing.

Table 6 shows the loss profiles and loss distribution of losses for several lines of business. The aviation insurance lines are Airports, Ground Handlers, Refuellers, Repairers and Services, Subcomponent Manufacturers and Prime Manufacturers. The books of business are quite different in size ($F, for example, is substantially larger than other lines) and that may unbalance the analysis. Also, the assumption of homogeneity within each class is more acceptable in some lines than others. However, for consistency the same analysis has been adopted for each line. Also shown is the aggregate limit that is purchased for each line.

The characteristics of the “average account” are captured in the graphic of Figure 4. Notice that each line has a different shaped tail. Refuellers (yellow triangles) clearly show a rather low median level of losses (see intercept at the top of the graph). However, clearly this line has a propensity to much higher loss levels relative to the median, than the others in the graph. As will be obvious, such a fat tail ought to reflect itself in capital needs and minimum prices.

At the line level, diversification by account produces the risk profiles in Table 6. Table 6 shows the level of loss to be expected for each line at various levels of probability. Thus at the 1:10 level, Airports can be expected to have a loss of 1.75 x $A million; Subcomponent Manufacturers can expect losses of 1.78 x $E million; Prime Manufacturers can expect 1.8 x $F million. Taken all together, if each line was capitalized to a 1:10 level the aggregate capital

![Fig 4]

RISK PROFILE OF 5 LINES, GENERATED FROM PROFILE OF "AVERAGE" ASSURED
required would be \(1.80 \times (A+B+C+D+E+F)\) million.

However, airport losses may be independent of prime losses so that capitalizing a whole book to the 1:10 level would require an amount of \(1.54 \times (A+B+C+D+E+F)\) million. Table 7 shows the effect of capitalizing the whole book at specific levels compared with separately capitalizing each line at that same level. In other words, it shows the effect of diversification. Clearly, \(0.26 \times (A+B+C+D+E+F)\) million less capital is needed because of the diversification effect at the 10% level. This is a 15% savings in capital. At more remote probabilities the savings are even greater. At the 1:100 levels the savings is 30%.

**Minimum capital for the book**

Using the same methodology for calculating capital as described in the Ground Handlers stand-alone example, the premium and capital necessary to accommodate a 1:10 loss to the whole book are, respectively, \(1.1246 \times (A+B+C+D+E+F)\) million and \(1.246 \times (A+B+C+D+E+F)\) million. The details are laid out in Table 8. Essentially, if a \(1.54 \times (A+B+C+D+E+F)\) million loss occurs (90% event), premium will cover \(1.1246 \times (A+B+C+D+E+F)\) million of the loss but there will be a “bite” to capital of \(0.4154 \times (A+B+C+D+E+F)\). If that bite is to be no more than 1/3 of capital, then capital must be \(1.246 \times (A+B+C+D+E+F)\) million.

Table 8 also shows that capital amount reallocated back to the individual lines. And here analysts have a choice of techniques. Theory is not much of a guide. If \(1.246 \times (A+B+C+D+E+F)\) million is the whole book capital what percentage should be allocated to the Airports...
line and others? One way to allocate the total capital is to do it proportionate to standard deviations for each of the lines. Another way is to allocate proportionate to the individual line 1:10 risk levels. These alternative possibilities are shown in Table 9.

In what follows we have chosen to use percentages proportionate to standard deviation. One clear difference shows up for Refuellers.

Remember, because of the long tail associated with that line, the Refuellers’ standard deviation is quite high. That effect does not show up if we had allocated at the 1:10 level. And yet, it is important that the lines causing the greatest exposure to capital should require the greater amount of capital. Clearly, the Prime Manufacturers’ line dominates the loss possibilities and it should require the greatest amount of capital. It should also carry the greatest share of the load. Those features are demonstrated in Table 10.

**Minimum price for limit**

Given the allocation of load that should be born by each line to achieve a minimum return on equity, we are now able to calculate the minimum premium each of the lines. Table 10 shows the aggregate premium by each line. Dividing the minimum premium target by the limits purchased would give the minimum cost of limit in basis points. The aggregate limits purchased in each line are not detailed here so we resist the temptation to take the final calculation step.
Airports show a minimum premium of 1.103 x $A million. Prime Manufacturers show a minimum of 1.121 x $F million. The difference results from two features. First, Prime Manufacturers experience much more mean or expected losses relative to Airports. The experience components of Airports and Primes are, respectively, $A million and $F (where F > 7 x A). Second, Primes minimums are much higher than other lines because the Primes contribute much more volatility to the book. Primes use more capital, carry more load, because they contribute more volatility.

Refuellers carry an even higher relative load, as they should. The load for Refuellers is nearly one third of the minimum total premium. Those privy to the limits purchased by Refuellers know that it is exceptionally high. On average, Refuellers account for only 2.2 % of book losses; however, they purchase close to 11% of known limits purchased. This may be partly due to the fact that the insureds themselves realized the skewness of their exposure. One consequence may be that the load may have to be spread over a disproportionately large limit and the price in rate on line terms may appear low. Perhaps limit was given away too cheaply in the past and a keener future pricing might result in lower overall purchases.

**Implementation**

Assuming the data for on line volatility is correct and purchased limits on liability is available, the minimum rates on line analysis will give a range of appropriate prices. Regrettably, the science may stop there. Art or at least business acumen is necessary to complete the pricing task.

Several things must be borne in mind in that final analysis. First, the objective of insurers is not to make a pre-tax, pre-investment income, pre-expense profit of just 10%. Nor will it be desirable to charge all accounts at the minimum acceptable level. Many will have to be charged more than the minimum. Finally, clients generally do not want to bear all the costs of their own (unlucky?) experience. Some sharing of experience (mutualization) may be desirable.

The precise premium to charge every account will therefore be a function of experience and strategic judgment. It will also depend very much on competitor actions and the competitive environment. A discretionary scheme for minimum charges that is consistent with required returns must nevertheless be established.

Ideally, the application of these minimums would then run as follows. Client X should be charged an amount given by his own loss experience and the insurers target rate of return on capital. That may be determined by some optimizing pricing formula such as herein. However, the premium for new clients with loss free records who seek large limits should never be less than the type of discretionary minimums suggested above.

**Discussion and Concluding Remarks**

Nearly all the figures in this analysis are subject to challenge. The exact numbers here, while reasonable, are not final nor are they intended to be replicated in actual decision environments. The purpose of the paper is to...
suggest an approach to answering what on the surface is a seemingly simple question. What is the minimum premium that should be charged for covers that have relatively large limits of exposure?

As an illustration of the importance of precise numbers, astute readers will have noticed that the final step in the calculation procedure is a relationship of “load” to “limits purchased”. The estimation of total limits purchased is critical to the final basis point ratio. The empirical evidence of what has been purchased in the past is, however, no doubt a function of what has been charged for limit in the past. A change in the charging procedure will change the limit purchase decision. This is an outcome to be desired since we want the client to respond to appropriate economic signals. However, over time the client and insurer may have to iterate to the exact ratios. For example, if the charging of a minimum premium caused insureds to reduce limits purchased by, say, one third, the minimum premium the following year may have to be raised. Not by the full third, since limits of exposure would have been reduced. However, some insights into the “elasticity” of the limits purchased decision would make the analysis more precise. What is obvious from the data is that Prime Manufacturers buy big limits in absolute terms, but buy smaller amounts relative to their own volatility than is done in other lines. This may be due to client insight into their exposures. It may be due to regulation. It may be due to cost. Primes are often given capacity charges for their large limits, which while not as large as suggested here, may be enough to elicit economic purchase decisions not evidenced in the actions of smaller lines where capacity charges are not always evident.

Our analysis has focused on minimum charges that emanate from the supply of capital perspective. Using purchased limits represents demand as static, and that needs to be improved in future analysis.

One final comment involves diversification credits. Some single digit minimums might result from the recommended analysis. In part this results from the fact that each line diversifies another and that capital devoted to non-aviation capital may be called into play in times of stress. While this leads to estimates of minimums lower than is seen in the stand-alone private market, why does the bond market not give itself more diversification credits? After all, lenders (bond buyers) can diversify between different corporations, and certainly large portfolios can support bond losses with profits from other securities. Why do these diversification possibilities not lead to bond spreads below the 20+ basis points seen in the market? For whatever reason, private markets resist giving themselves too much credit in that direction. Diversification credit without analysis is a slippery slope. As our numbers show, given big enough diversification credit the calculated minimums can be very low indeed. And who is to say, absent a market, what frequency of loss and what severity of loss (capital bite) is ultimately acceptable. Certainly, a review of aviation insurance results over the past decade shows that the insurers appear to have accepted too high a frequency of loss. Perhaps it is appropriate to close this analysis then by cautioning that insurers would do well to keep on the “scientific” side of the angels by paying close attention to capital market comparatives. They can be sure that providers of capital, who can move or withdraw capital at will, will be doing the same.

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1 Any errors in interpretation of the data or calculation herein are solely the responsibility of the author.

2 The two equations are: (Premium - 90% Loss) > 1/3 Capital, and Premium- Expected Loss>10% Capital. Solve for Capital and Premium in terms of Expected Loss and 90% Loss. See also Table 8.

3 It is of interest to note that the line load is also equal to about approximately 18% of Standard Deviation. (Compare Table 6 and Table 10.) If the target rate was 20% instead of the minimum of 10% the load would equal 36% Standard Deviation. This is similar to the well known Kreps price.